Topic Study Group 1

Mathematics Education at Preschool Level

Marja van den Heuvel-Panhuizen[1], Angelika Kullberg[2], Ineta Helmane[3], and Xin Zhou[4]

1. Aims of the TSG

TSG-1 is about the foundations of learning mathematics and the contexts in which the first steps are taken towards achieving mathematical understanding. The aim is to share and discuss contemporary research on early childhood mathematics learning and teaching and their theoretical and methodological frameworks. TSG-1 involves research on children's mathematical development from birth until entering formal schooling in first grade (children up to 6). The nurturing of this development can take place in care centers, preschool, and kindergarten, and at home.

Although it is currently widely accepted that the development of mathematical skills in the early years is essential for later mathematics learning, it is not so obvious what mathematics should be fostered in young children. Mathematics as a subject has traditionally been considered above the preschool and kindergarten levels. Moreover, researching young children's mathematical understanding has for a long time been a privilege of psychology and pedagogy. These sciences have provided much knowledge about conditions and variables that influence children's mathematical development but do often not consider very deeply the mathematics that is, or has to be, developed by young children and generally do not cogitate about why certain mathematical competences are important or what activities are crucial to stimulate the development of these competences. To gain a better insight in this what aspect of mathematics education at preschool level, we invited contributions from the didactics of mathematics, but also for example from (neuro-) cognitive, developmental, socio-cultural and other approaches to the learning and fostering of young children's mathematical understanding. TSG-1 intends, from multiple perspectives, to contribute

[1] Faculty of Education and Arts, Nord University, Norway and Freudenthal Institute, Utrecht University, Netherlands. E-mails: m.vandenheuvel-panhuizen@nord.no; m.vandenheuvel-panhuizen@uu.nl

[2] Department of Pedagogical, Curricular and Professional Studies, University of Gothenburg, Gothenburg, Sweden. E-mail: Angelika.Kullberg@ped.gu.se

[3] Department of Preschool and Primary Education, University of Latvia, Riga, Latvia. E-mail: ineta.helmane@gmail.com

[4] Department of Early Childhood Education, East China Normal University, Shanghai 200062, China. E-mail: xzhou@pie.ecnu.edu.cn

to the improvement of knowledge and understanding of issues that early childhood mathematics education encounters in different contexts and come eventually with proposals for advancing research, development and practice in mathematics education at preschool level.

To achieve this TSG-1 we invited submissions of substantial research-based theoretical or empirical contributions within the following four subthemes:

1. Unpacking early childhood mathematics. Opening up the thinking about the mathematics content (broadly interpreted as knowledge, skills, conceptual understanding, mathematical reasoning and attitude) to be fostered in young children. What mathematics is worth to be developed? What mathematics anticipates future learning and opens the road to future learning?
2. Pedagogical and didactical approaches in early childhood mathematics education. What are meaningful learning environments for young children in a school setting or home environment? What tools, including manipulatives and technology, supports early mathematics learning? How can play and story reading be used? In what way can learning environments for young children be improved by embodiment theories on learning?
3. Assessing mathematical understanding in early childhood. How to get a better understanding of young children's mathematical development?
4. Preparing early childhood educators to foster children's mathematical development. How can professional development provide appropriate support and flexibility to allow teachers, care-givers and parents to develop new knowledge and understanding about mathematics education for young children?

1.1. Submissions

The 2021 round of submissions ended up in 17 accepted papers from 14 countries (North America: 3; Asia: 5; Europe: 9; Australia: 2).

1.2. Sessions

The TSG met during three days. In the first session, the TSG chair Marja van den Heuvel-Panhuizen, and the rest of the organizing team, described the organization of the sessions. Before the conference all papers and one power point slide for each paper were sent to the participants. The sessions were organized around two themes. The first session was on the theme Investigations of children's learning, whereas the two last session was on the theme Investigations of children's learning environment. Each paper was presented and discussed during 10 minutes. At the end of each session there was a joint discussion for 25 minutes on topics related to the papers as a whole and developments in the field. The list of papers and order of presentation are shown in Tab. 1.

Tab. 1. List of papers presented

Paper and author(s)
[1] Application of number line estimation strategy for 5–6 years old children: Effect of reference point marking. ***Xiaoting Zhao*** *and Xiaohui Xu* (China).
[2] Unraveling the quantitative competence of kindergartners. ***Marja van den Heuvel-Panhuizen*** (Norway) *and Iliada Elia* (Cyprus).
[3] Insights about constructing symmetry with 5-year-old children in an artistic context, ***Yuly Vanegas***. *Carla Rosell and Joaquin Giménez* (Spain).
[4] Kindergartners' use of symmetry and mathematical structure in representing SELF-portraits. ***Joanne Mulligan*** *and Gabrielle Oslington* (Australia).
[5] Investigating evidence of girls' and boys' early symmetry knowledge through multiple modes of assessment. ***Nicole Fletcher***, *Diego Luna Bazaldúa, and Herbert P. Ginsburg* (USA).
[6] 4-Year-olds children's understanding of repeating patterns: A report from China. ***Fang Tian*** *and Jin Huang* (China).
[7] Investigating how kindergartners represent data with early numeracy and literacy skills through a performance task. ***Insook Chung*** (USA).
[8] Counting activities for young children: Adults' perspectives. *Dina Tirosh, Pessia Tsamir,* ***Ruthi Barkai***, *and Esther S. Levenson* (Israel).
[9] Asking early childhood teachers about their use of finger patterns. ***Miriam M. Lüken*** *and Anna Lehmann* (Germany).
[10] Performance expectations in the area of "Shapes and Spaces" of early childhood educators in an international comparison. ***Catherine Walter-Laager*** (Austria), ***Manfred R. Pfiffner*** (Switzerland), *Xin Zhou* (China), *Douglas H. Clements* (USA), *Julie Sarama* (USA), *Linh Nguyen Ngoc* (Vietnam), *Lars Eichen* (Austria), *and Karoline Rettenbacher* (Austria).
[11] Mathematics in play. ***Ronald Keijzer***, *Marjolijn Peltenburg, Martine van Schaik, Annerieke Boland, and Eeffe van der Zalm* (Netherlands).
[12] Does preservice teacher training change prospective preschool teachers' emotions about mathematics? ***Oliver Thiel*** (Norway).
[13] Bishop's (1988, 1991) mathematical activities reframed for pre-verbal young children's actions. ***Audrey Cooke*** *and Jenny Jay* (Australia).
[14] When math meets games — The active construction of children's core mathematics experience in games. ***Jianqing Wen*** (China).
[15] Analysing a Danish kindergarten class teacher's instructional support in mathematics with the tool Class. ***Birgitte Henriksen*** (Denmark).
[16] Mathematical learning environments in Norwegian ECEC child groups. ***Øyvind Jacobsen Bjørkås***, *Dag Oskar Madsen, Anne Grethe Baustad, and Elisabeth Bjørnestad* (Norway).
[17] "More Gooder": children evaluate early numeracy apps. ***Ann LeSage*** *and Robyn Ruttenberg-Rozen* (Canada).

2. Conference Themes

The studies discussed in TSG-1 involve mainly research on children's mathematical development in the years before they enter in formal schooling in first grade. The nurturing of this development can take place in various environments: care centers, preschool, kindergarten, and at home. The 17 submitted papers to TSG-1 was divided in the categories: “Investigations of children’s learning” (papers [1]–[7]) and “Investigations of children’s learning environment” (papers [8]–[17]).

The papers in the first category are all based on data collected from children. For several mathematical content areas and competences it is investigated what children are capable of. The papers of the second category are based on data collected through observing classrooms and interviewing early childhood teachers and educators, prospective preschool teachers, and other adults. Interestingly, in one study the learning environment was also investigated by interviewing children themselves. In this second category are also two papers which have a more theoretical stance. One is proposing a revision of a framework for mathematical activities and the other is recommending the use of mathematical games in kindergarten.

2.1. Investigations of children’s learning

The collection of papers in this section addresses mathematical competences in the domain of early number, symmetry, patterns, and representation of data.

With respect to early number, one of the Zhao and Xu[1] investigated children’s competence in making estimations on the number line. This is a topic that is not everyday dealt with in kindergarten classes. By eye-tracking technology the study showed that the ability of kindergarten children (aged five to six) of making estimations on a 0–10 number line can be effectively improved by using a midpoint marker instead of a marker at every quartile.

Heuvel-Panhuizen and Elia[2] aimed to unravel the composition of the quantitative competence of kindergartners. By analysing data from a collection of paper-and-pencil items it was revealed that in addition to counting, subitizing, and additive reasoning, also multiplicative reasoning belongs to this early number ability. Furthermore, an implicative analysis at item level showed that in general, multiplicative reasoning and conceptual subitizing items were found at the top of the implicative chain, counting and perceptual subitizing items at the end, and additional reasoning items in the middle.

Three studies investigated the development of the notion of symmetry in kindergartners. In the study[3] of Vanegas et al. a sequence of 16 symmetry-related activities was developed in the context of art work. In these activities kindergartners had to work with various axes of symmetry. The authors found that the designed sequence can constitute a hypothetical path by which children in early childhood education can progress in their learning of symmetry.

Mulligan and Oslington[4] contributed the second study on kindergartners’ competence in symmetry, which looked for an alternative for the often used “butterfly” pictures. To make the context more meaningful for the children they had to work with drawings of their portraits which they had to analyse for features of line symmetry and mathematical structure. The authors found that over thirty percent of children represented explicit structural features such as equal spacing, congruence, partitioning and alignment of facial features. The third symmetry study[5] by Fletcher et al. explored the assessment of early symmetry knowledge. In the study an intervention with symmetry software took place in which first and second grade children were taught reflection, translation and rotation. After the intervention the children were assessed by a paper-and-pencil test and by interviewing them. The authors found that children who reached higher scores on reflection and translation tasks, in the interviews also provided explanations indicating conceptual understanding of the symmetric transformations. The similar relationship was reported for girls and boys.

Recognizing and being able to work with patterns is considered a vital element of young children's mathematical development. To know more about children's understanding of patterns, Tian and Huang[6] investigated in a sample of 134 four-year-olds preschool children how able they are in solving tasks on repeating patterns. The results showed that the children could fill and expand repeating patterns, but also difficulties came to the fore in the abstraction of the pattern, especially in identifying the unit of a repeating pattern.

The last content domain that is reported in this section is the representation of data. Chung[7] describes a study in which it was investigated whether kindergartners (aged five-six) can sort and group objects, identify the quantity of each group of objects, and then can draw pictures or write names and numbers to organize and present the data. One of the results of the study is that half of the 35 children involved failed to represent the quantities using numerals and pictures.

2.2. Investigations of children's learning environment

The papers in this section lift in different ways the veil of the conditions and circumstances in which the early learning of mathematics can come about. To gain knowledge about this, in most studies data were collected by interviewing early childhood teachers. In the study[8] by Tirosh et al. a broader response group was surveyed and adults (not being preschool teachers but including grade school and high school teachers, psychologists, occupational therapists, engineers, municipal workers, and accountants) were asked what types of activities they perceive as the ones that can promote numerical skills. Many participants suggested counting objects. Sub-skills such as counting forward from some number other than one or focusing on one-to-one correspondence, were less mentioned.

In Lüken and Lehmann's study[9], when 23 early childhood teachers were asked about their use of finger patterns in their daily interaction with children it was found that they all use finger patterns in a variety of everyday (such as age/birthday, finger games, board games) and mathematical contexts (verbal counting, object counting, referring to quantities or number signs, and when calculating). The frequency and type of used finger patterns varied among the teachers. Only four teachers used finger patterns doing calculations. Two of them used the fingers in a dynamic way and two in a static way. No more then ten teachers used finger patterns as a visualization to help children develop an understanding of numbers.

Because what early childhood educators think about the mathematical abilities of their children may influence the learning environment they offer to them, an international study[10] by Walter-Laager et al. was set to investigate the performance expectations of early childhood educators in five countries. The focus was on shapes and space. The data of 1343 early childhood educators revealed that the expectations for this content area were more accurate in Austria and Switzerland than in China, Vietnam and the USA. Also, the estimations for 3–6-year-old children were more appropriate than those for the 1–3 year olds.

In addition to learning through focused activities, young children's learning of mathematics also takes place to a large extent through free play. In Keijzer et al.'s report[11], to figure out what interactions between preschool/kindergarten teachers and preschoolers (2 to 6 years) can be considered as useful for stimulating young children's language and mathematical development a professional learning community (PLC) was set up consisting of preschool and kindergarten professionals and researchers. Based on discussions held within PLC-meetings and the analysis of the mentioned interaction characteristics three guidelines for interactions were identified that can stimulate children's mathematical development during children's spontaneous play: observing (understand the child's interest and feelings), connecting (confirm what the child is playing) and enriching (cooperatively construct mathematical meaning).

Preschool teachers' positive feeling about mathematics is a determining factor of the quality of the early childhood learning environment. Therefore, in Thiel's longitudinal study[12] with an experimental pre-test post-test control group design, it was investigated whether and how a preservice teacher training can change prospective early childhood teachers' emotions about mathematics. The study was carried out with full-time and part-time teacher students. Only the part-time students showed after the training an increase in mathematics enjoyment and a reduction of mathematics anxiety. For almost all the part-time students the lessons at the university were the most important reason of this change. For only half of the full-time students this was the case, while 35% indicated that it was the five-weeks practical period they spent in an early childhood institution.

In the two following papers, instead of an empirical approach, the learning environment is considered from a theoretical point of view. Cooke and Jay[13] discussed with what mathematics young, pre-verbal children might be engaged. The authors used for this Bishop's framework of the six mathematical activities which are fundamentally mathematical: counting, locating, measuring, designing, playing, and explaining. By reframing each of these activities by putting the focus rather on actions than on language, the framework is made appropriate for pre-verbal children and may provide assistance in identifying the mathematical thinking that is evident in pre-verbal children's actions. Wen[14] focuses on games as the basic form of activity for preschool children and describes the mathematics that children can meet in games and through which they can achieve the ability to think mathematically. Questions to be answered are how the gameplay and the core mathematics experience are related and how the fun of games can be combined with the effectiveness. The paper continues by giving examples of teachers playing games with children and children playing alone or cooperatively.

A tool to measure the quality of the early childhood learning environment in a standardized way is the Classroom Assessment Scoring System (CLASS). With this tool, among other things, the given instructural support can be investigated with respect to three dimensions: the development of concepts, the quality of the feedback and the language modelling. Henriksen[15] used this tool in a kindergarten class and analyzing the classroom interaction in an observed lesson. It was revealed that there was a low score on Instructional Support: the teacher did not prompt children to explain their strategy, did only focus the feedback on the correctness of the answers, and asked mostly close-ended questions. By proving this information, the tool can give indications in what way the teacher may develop.

In a large national study[16] by Bjørkås et al., the quality of the learning environments in the child groups of Early Childhood Education and Care centers were investigated by means of data based on observations with the Infant/Toddler Environment Rating Scale —Revised and the Early Childhood Environment Rating Scale — Revised. The focus in these observations was on the learning area “Number, Spaces and Shapes”. In addition, questionnaires were used to collect from directors of ECEC centers. A comparison of the results with a study done some seven years ago showed that the centers worked more systematically on this learning area. However, the quality of the learning environments as measured through the observations varies greatly and are to a large extent qualified as inadequate. For example, most of the centers only provided one kind of blocks on a daily basis, giving little opportunity for children to investigate different kinds of properties of space and shape.

In the final paper[17] by LeSage and Ruttenberg-Rozenan an alternative research perspective was chosen. In this study children themselves was given a voice when investigating the quality of the early childhood learning environment. The focus was on the quality of educational software. In particular five early numeracy apps were investigated, which were uploaded onto the classroom iPads. Data from 12 children (4 to 6-year-olds) were collected through multiple sources, including observations, interviews and videotaped child-led ‘tours’ of their favorite apps. As criteria for good apps were identified the quality of the game experience (frequent positive verbal reinforcement and earning rewards) and the autonomy in making choices.